

Application of the D^2 -Law to Determine Time Evolution and Burn-Out Time of Evaporating Biodiesel Spray Drop-Size Distribution

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Abstract: This paper applies the D^2 -law to a group of heterogeneous size of biodiesel drops in order to determine the time evolution of the drop-size distribution and its burn-out time. Experiments to determine initial drop distribution data, used for predicting the evaporation time, have been performed on a biodiesel spray issuing from a small scale industrial burner. The measured drop-size distribution has been first transformed into a dimensionless form, called mass-based drop-diameter distribution, before its time dependent distribution and the evaporation time are calculated. It is found that the D^2 -law, originally applied for determining the evaporation time of a droplet, can be adapted well to calculate the time evolution of the drop-size distribution and its evaporation time. The dimensionless evaporation time is shortest for a group of mono-sized drops and prolonged for a drop-size distribution provided that they are at the same initial mean diameter. Furthermore, if the initial dimensionless mass distributions contain high proportion of small drops; it is prone to evaporate faster than those containing larger drops. The dimensionless evaporation times obtained from this study are the analysis results of the drop-size distribution along the line-of-sight measurement data. In this work, the variations of the volume concentration along the line-of-sight are omitted.

Keywords: Biodiesel spray, Burn-out time, D^2 -law, Dimensionless mass-based drop-diameter distribution, Time evolution mass-based drop-diameter distribution.

1. Introduction

Biodiesel is a pertinent biofuel for Thailand and many other countries. It has already been used in the form of diesel-biodiesel blend B2 and B5 for automobiles and trucks. In the industrial sector, it has also been used as diesel and fuel oil substitute in burners and furnaces for heating applications. Most of the existing technologies to convert liquid fuel to power, particularly the spray combustion technologies, have been designed for regular diesel fuel combustion conditions. The replacement with biodiesel or biodiesel blends in a system originally designed for the diesel usage results in an off-design operating condition, resulting in lower thermal performance and sometimes combustion instability.

A fuel spray comprises hetero-sized drops, which can be characterized in the form of drop-size distribution. Comparative measurements of drop-size distributions within the diesel and the palm oil-based biodiesel sprays issuing from the same nozzle and at the same injection pressure have been carried out by same workers. For example, it has been found that the mean diameters of the drops of the biodiesel are larger than those of the diesel [1]. Dimensionless mass-based drop-diameter distribution model based on maximum entropy formalism has been developed and used to fit the distributions obtained from the measurement data of diesel and biodiesel sprays. It has been shown that the mass-based drop-diameter distribution model, which is a non-dimensional model, can very well be used to fit the measurement data [2].

When droplets are to be burned in a combustion device, their burn-out time is an important parameter for the design and operation of the device. Experimental setups to determine evaporation rate of a single droplet within elevated temperature environment can be found, the classical ones being those of Kuo [3], Spalding [4], and Ranz et al. [5], and the more recent ones are those of Faeth et al. [6], Nakayama et al. [7] and Knubben et al. [8]. Direct measurements of time evolution of the drop-size distribution using laser techniques are rare. This is because of the difficulties on determining of temperature and size of drops

in the elevated temperature and evaporating environment. Although there are measurement reports on related studies [9- 10], some challenging problems remain unresolved, notably the simultaneous measurements of temperature and drop-sized distributions of group of evaporating drops.

Theoretical studies on the prediction of burn out time of group of drops are also limited. One that is often cited is the theory describing the burn out time using the time evolution of mass based drop-size distribution proposed by Williams [11]; in which the burn-out time was defined by the time that 99% of mass was consumed.

This paper tries to find a means to predict the evaporation time of the hetero-sized drops within the biodiesel spray generated from a small scale industrial burner. A simple evaporation model, originally used for single droplet, the D^2 -law [3-5], has been adapted to the hetero-sized drops within the spray in order to calculate the time evolution of the drop-size distribution and then to predict its burn out time.

2. Experimental

2.1 Experimental setup

Malvern Spraytec instrument is used in this study to measure drop-size distribution of the biodiesel spray. The measurement is based on laser diffraction technique. A schematic diagram illustrating the experimental setup is shown in Fig. 1. The measurement volume for this technique is the entire laser beam containing drops of hetero size. Laser from the light source scatters from drops; and the forward diffracted light being focused by the Fourier lens onto the ring detectors. These measured intensities are then converted into a volume-based drop-size distribution using the embedded software within the Malvern instrument. The technique takes into account the forward diffraction and refraction scattering effects. Unscattered light, on the other hand, is focused onto the central detector, giving information on obscurity of the measurement volume.

The nozzle used in this study is a pressure swirl type giving a solid cone spray. The injection pressure used is 12 bar giving a biodiesel volume flow rate of 3.8 liter per hour. The site of the measurement is on the horizontal plane at 7 cm below the nozzle exit. There are seven line-of-sight measurements, scanning from the center of the spray ($y = 0$) to the outer edge of the spray ($y = 6$ cm); each measurement being 1 cm apart – a distance equal to the laser beam diameter. The viscosity, density and surface tension coefficient of biodiesel used in this study are 5.7 cSt, 875 kg/m³ and 29.9 mN/m, (at 40°C) respectively.

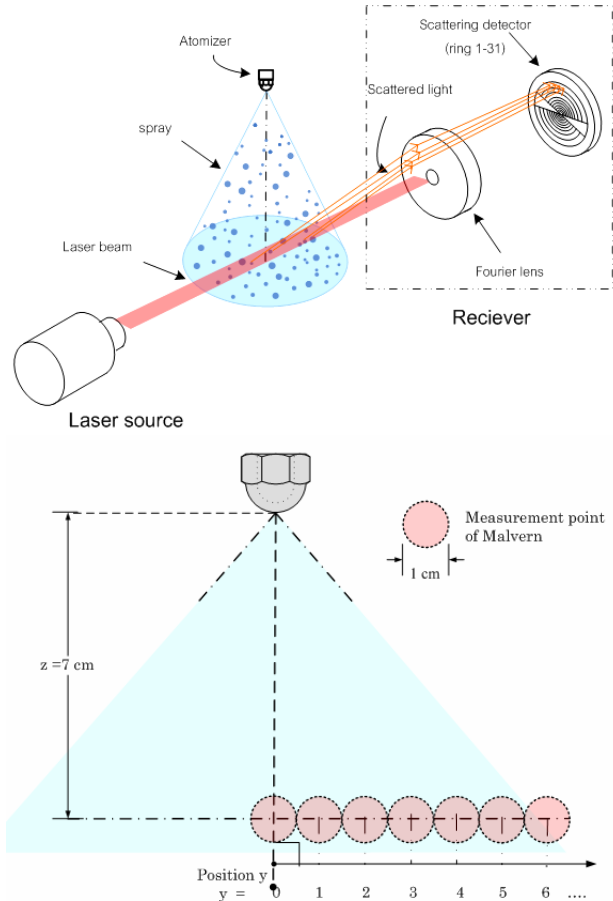


Figure 1. Schematic diagram of drop-size distribution measurement.

2.2 The measurement of mean diameter

The Malvern Instrument reports the volume fraction of drops along the line-of-sight of the laser beam. The volume fraction is the ratio of the volume of drops, ΔQ_i , within the diameter class i with width, ΔD_i , to the total volume of drops, Q_{Total} , from all diameter classes ($i = 1$ to M) within the measurement volume. The volume fraction distribution can be used to obtain different type of mean diameters. The De Brouckere mean or D_{43} , which is used commonly in combustion applications, has been applied in this model. This mean diameter is defined by

$$D_{43} = \frac{\sum_{i=1}^M D_i \Delta Q_i}{Q_{Total}} \tag{1}$$

where D_i is the median diameter of class i .

2.3 Dimensionless drop-size distribution

To reduce the number of variables and to be independent of system of unit, the dimensionless diameter is used in this work. The diameter D_i and width ΔD_i of each class of the volume

fraction distribution are divided by the mean diameter D_{43} of the distribution to obtain the dimensionless diameter denoted by ξ_i and the dimensionless class width $\Delta \xi_i$; i.e.:

$$\xi_i = \frac{D_i}{D_{43}} \quad ; \quad \Delta \xi_i = \frac{\Delta D_i}{D_{43}} \tag{2}$$

The volume fraction, on the other hand, is expressed in the form of mass fraction. This is more appropriate as far as evaporation or combustion of the group of drops is concerned. Since liquid fuel density is constant, the mass and the volume fraction distributions are the same. Therefore, the dimensionless mass-based drop-diameter distribution is identical to the volume-based drop-diameter distribution, i.e.:

$$\frac{\Delta w_i}{w_{Total}} / \Delta \xi_i = \frac{\Delta Q_i}{Q_{Total}} / \left(\frac{\Delta D_i}{D_{43}} \right) \tag{3}$$

where w_{Total} represents the total liquid mass and Δw_i the mass represented by the class i droplets, i.e., those with diameter D_i . The dimensionless mass-based drop diameter distribution shown in the above equation is in turn used as the initial distribution for analysis of evaporation process.

2.4 Evaporation of hetero-sized droplets

When a single fuel droplet evaporates under a quasi environment, either with or without combustion, its size against the evolution time is commonly expressed by the D^2 -law [3]; i.e.:

$$D^2 = D_0^2 - \lambda t \tag{4}$$

where D_0 is initial diameter, λ is the evaporation constant and t is the evolution time.

The burn out time, t_b , is reached when the diameter of the droplet becomes zero. The D^2 -law can be written in dimensionless form, using the mean diameter, D_{43} . It becomes:

$$(\xi_{i,\tau})^2 = (\xi_{i,0})^2 - \tau \tag{5}$$

where $\tau (= \lambda t / D_{43}^2)$ is the dimensionless time. It is important to mention here that the mean diameter D_{43} used in Eq. (5) is kept constant at any time and equal to the one at the initial time, i.e., $D_{43} = D_{43}(t=0)$.

When a group of drops evaporates under elevated temperature environment, the number of drops having the same diameter does not change with time although their sizes are decreasing. This is illustrated in Fig. 2. The concept schematically shown in this figure can be written as $\Delta N_i(t) = \Delta N_i(0)$ where ΔN_i represents the number of drops in class i . Based on this consideration, the time dependent mass-based drop-diameter distribution can be written as:

$$\left(\frac{\Delta w_{i,\tau}}{w_{Total}} / \Delta \xi_i \right) = \frac{(\xi_i^2 - \tau)^{3/2}}{\xi_i^3} \left(\frac{\Delta w_{i,0}}{w_{Total}} / \Delta \xi_i \right) \tag{6}$$

Note that the time dependent mass-based drop-diameter distribution (left hand side of Eq. (6)) is not normalized anymore,

i.e., $\sum_i \frac{\Delta w_{i,\tau}}{w_{Total}} \leq 1$, the equality being satisfied at $\tau=0$ only.

This is due to the fact that w_{Total} and $\Delta \xi_i$ are kept constant and equal to the total mass and to the dimensionless class width at $\tau=0$ respectively. This allows us to define the burn-out time τ_b as the

time at which $\sum_i \frac{\Delta w_{i,\tau_b}}{w_{Total}} = 0.01$, in agreement with Williams

[11]. Note that according to Eq. (5), the burn-out time of an ensemble of mono-sized droplets is $\tau_b = 1$.

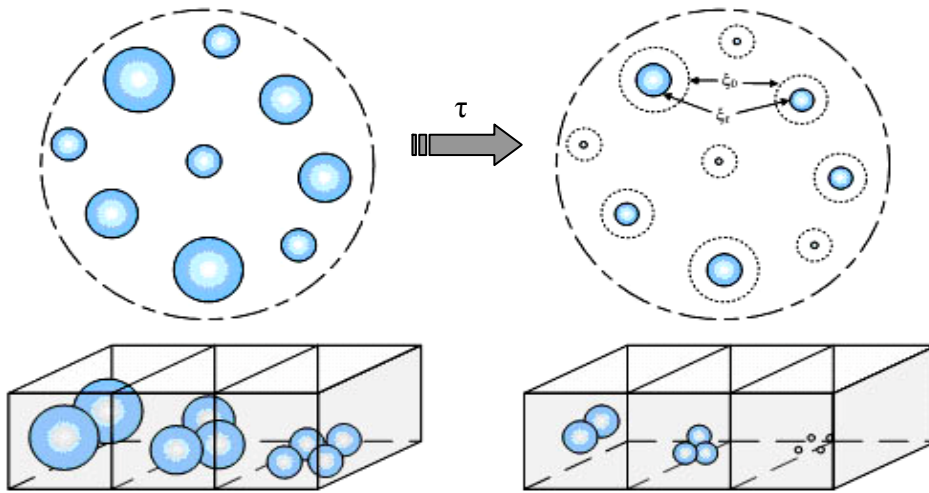


Figure 2. The evaporation concept of hetero size drops.

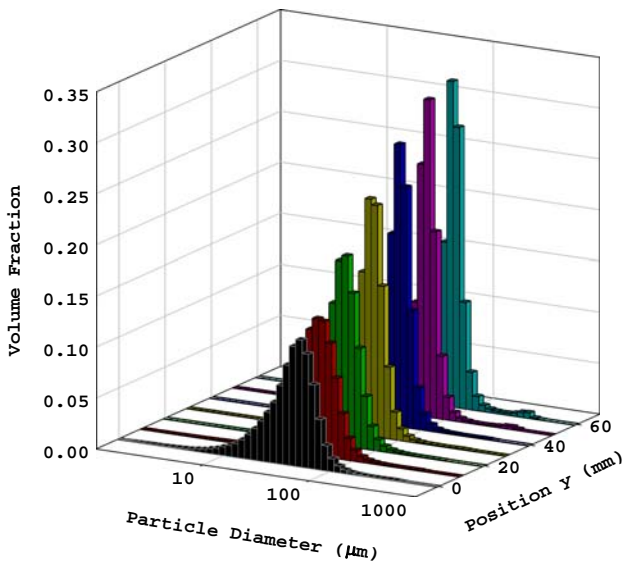


Figure 3. Volume fraction distributions at different line-of-sight measurements.

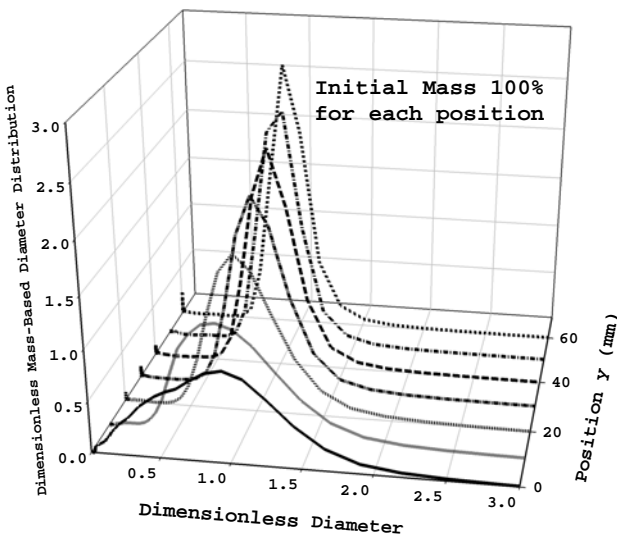


Figure 4. Mass frequency of different line-of-sight measurement at initial dimensionless time.

3. Results and discussion

3.1 Volume fraction distribution

The measurement results of volume fraction distribution are shown in Fig. 3. It can be seen in this figure that the distributions obtained are broader and the average diameters are smaller as the line of measurements get closer to the center of the spray. The mean diameters D_{43} obtained from the different line-of-sight measurement are 46.8, 47.8, 52.6, 57.9, 63.9, 70.9 and 77.6 μm from the center to the outskirts of the spray respectively. Diameter space of the volume-based drop-size distribution is nondimensionalized by D_{43} and the volume distributions are represented in the form of mass-based drop-diameter distributions as shown in Fig. 4.

3.2 Time evolution of the mass-based drop-diameter distribution

Volume fraction distribution data at position $y = 0$ is changed into dimensionless mass-based drop diameter distribution using Eq. (3). The dimensionless mass-based drop diameter distribution is plotted in Fig. 5 (the solid line). Since the diameter space is nondimensionalized by the mean diameter D_{43} the dimensionless distribution locates on the dimensionless diameter space of order one.

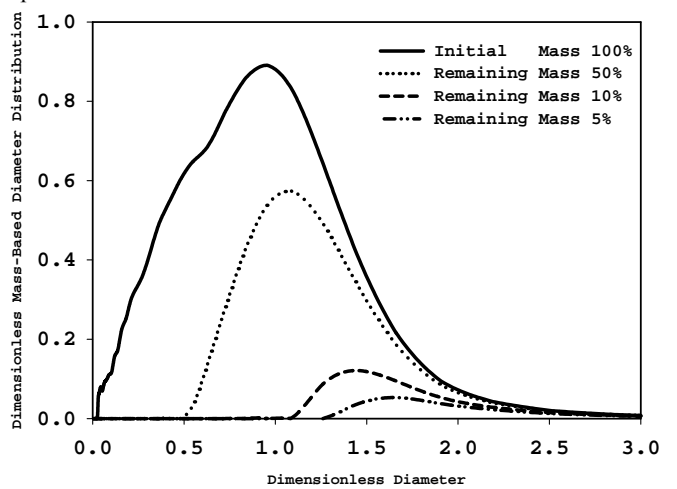


Figure 5. Time evolution dimensionless mass-based diameter distribution.

As indicated above, the integration of the mass-based distribution gives the ratio of mass at the evolution dimensionless

time τ to the mass at the initial time $\tau = 0$. Since the solid line represents initial dimensionless drop-diameter distribution, then the area under the curve of solid line (representing the mass fraction) is 1. When combustion (or evaporation) occurs, the D^2 -law is applied on the dimensionless mass-based distribution and the curve evolves with time. The dot, dash and center lines shown in Fig. 5 are the dimensionless mass-based drop-diameter distribution when the remaining mass ratio is at 50, 10 and 5% respectively. It can be seen in Fig. 5 that the small drops disappear first and large drops remain. It should be reminded here that the dimensionless mass-based drop-diameter distribution is plotted against the initial dimensionless diameter.

3.3 Evaporated mass ratio

The evaporated mass ratio against the evolution time of the above distribution (at the position $y = 0$) is calculated and plotted in Fig. 6. There are two time scales in this plot: the scale on the bottom x-axis is the dimensionless time scale and the scale on the top x-axis is the real time scale. Since at current state of the research, the value of the evaporation constant (λ) of the bio-diesel is not available then the magnitude of the real time (t) is scaled by λ .

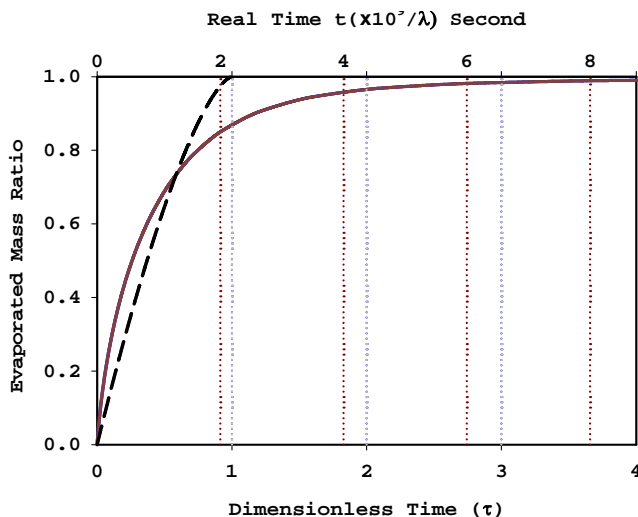


Figure 6. Evaporated mass versus real and dimensionless time scales.

The dash-line in Fig. 6 represents evaporated ratio for a group of mono-sized drops. It is shown in the figure that for the drop-distributions of the same mean diameter; broadening of the drop-diameter distribution delays its evaporation time. The dimensionless evaporated time is shortest (with the value of 1) when the group of drops is of mono-sized drop distribution. Fig. 7 shows calculation results of the dimensionless mass-based drop-diameter distributions (whose distributions at the initial time are shown in Fig. 4) at particular time after the evaporation. At the initial time, the distributions are broader and contain smaller mean diameter in the central spray region than in the outskirts of the spray. Later at the particular evolution time after the evaporation of the drops, the remaining mass ratio at the centre (= 10%) remains less than that at the outer edge (= 36%). It can be seen that when the initial dimensionless mass distribution contains high proportion of small drops, it is prone to evaporate faster. In the case where the fuel and the combustion conditions are similar (λ is a constant and independent of drop-size), the real burn-out time depends only on the mean diameter: the smaller the mean diameter the faster the real evaporation time of the distribution.

The fact that the burn-out time is a function of the mean diameter only is because this time has been defined from a typical

value of the remaining mass fraction i.e. $\sum_i \frac{\Delta w_{i,\tau_b}}{w_{Total}} = 0.01$.

Note that if w_{Total} is initially high, this condition can be reached even if a non negligible amount of liquid has not been evaporated. The burn-out time defined here is based on relative information on the evaporation process.

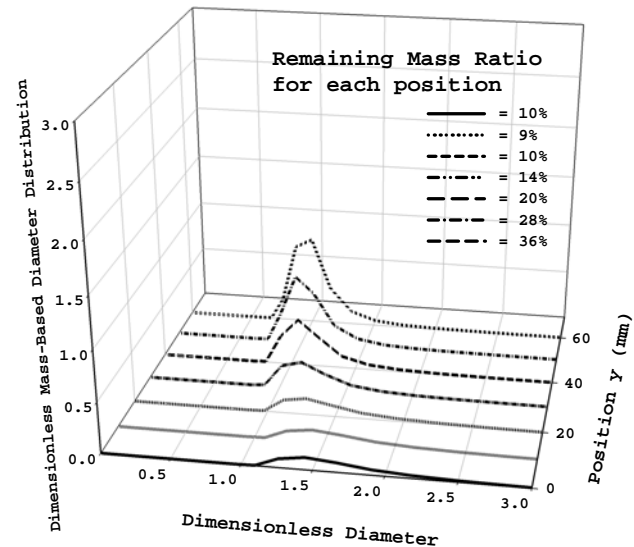


Figure 7. Mass-based diameter distribution of different line-of-sight measurement at a particular evolution of dimensionless time.

4. Conclusion

The classical D^2 -law for evaporation of a droplet can naturally be adapted to analyze a group of drops and their evaporation time. Integration of the distribution over the dimensionless diameter space at any evolution time, results in the remaining mass ratio at that time and the results are being used to calculate for the evaporation time of the distribution. It can be seen that the dimensionless evaporation time for the drops of mono-sized is one; it is prolonged for the drops of hetero-+ogeneous sizes. To calculate the real evaporation time from its dimensionless value, evaporation constant and mean diameter of the distribution are needed. The experimental data show that the distributions obtained are broader and the average diameters are smaller as the line of measurements get closer to the center of the spray; the evaporation times being faster for the groups of smaller drops.

The evaporation time defined within this paper is applied for the distribution of drops along the line-of-sight of measurement volume. The effects of concentration liquid drops along the measurement volume on the evaporation time are omitted in this study. Simultaneous reconstruction of the local drop-size distribution and the concentration within the biodiesel spray from their line-of-sight measurement data and their evaporation time are the subjects for future research.

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